יש k מחלקות, $p_1, \dots, p_k$ – סיכוי לראות מחלקה מסויימת.

$x_i \in \{0,1\}^d$ ו- $\Pr(X_i = x_i | z_i = j) = \prod_{k=1}^{d} \Pr(X_{i,m} = x_{i,m} | z_i = j)$

$$q_{j,m} \coloneqq \Pr(X_{i,m} = 1 | z_i = j)$$

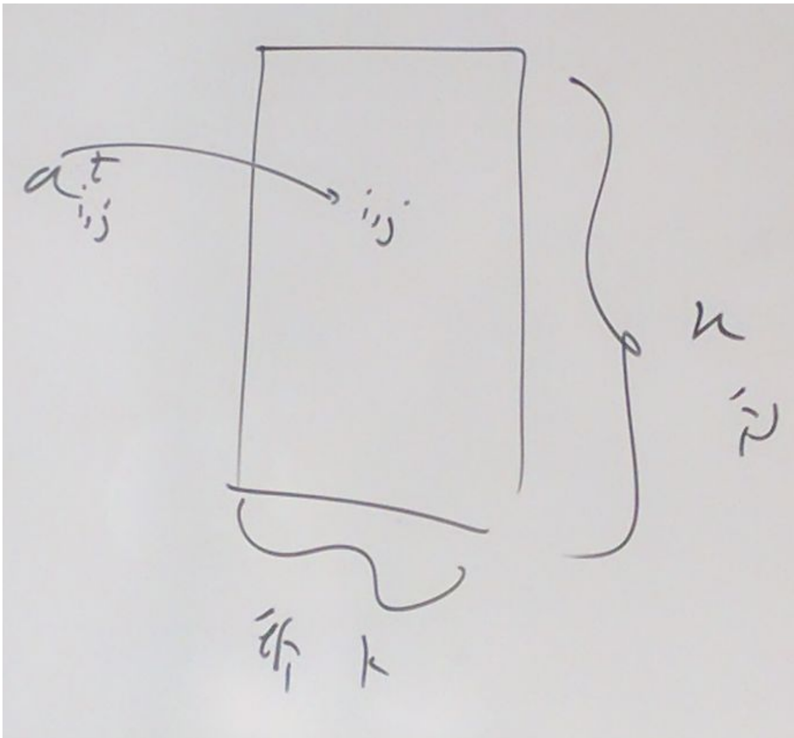
$$\hat{\theta} = (p_1, \dots, P_k, q_{1,1}, \dots, q_{1,d}, \dots, q_{k,d})$$

נתונות $x_1, \dots, x_n \in \{0,1\}^d$, רוצים למצוא את $\hat{\theta}$. נעשה EM.

**E-step-** נחשב את $a_{i,j}^t = \Pr_{\hat{\theta}_t}(z_i = j | X_i = x_i)$

$$= \Pr_{\hat{\theta}_t}(X_i = x_i | z_i = j) * \frac{\Pr_{\hat{\theta}_t}(z_i = j)}{\sum_{l=1}^{k} \Pr_{\hat{\theta}_t}(X_i = x_i | z_i = l) * \Pr_{\hat{\theta}_t}(z_i = l)}$$

$$= \frac{p_j^t \prod_{m=1}^{d} \left(q_{j,m}^t\right)^{x_{j,m}} \left(1 - q_{j,m}^t\right)^{1 - x_{j,m}}}{\sum_{l=1}^{k} p_{l,m}^t \left(q_{l,m}^t\right)^{x_{l,m}} \left(1 - q_{l,m}^t\right)^{1 - x_{l,m}}}$$



***M-step*-** נגדיר $Q(\hat{\theta} | \hat{\theta}_t) = E_{Z \sim \text{אפוסטרי}}\left[\log\left(\Pr_{\hat{\theta}_t}(X_1 = x_1, \dots, X_n = x_n, z_1, \dots, z_n)\right)\right]$

$$= \sum_{i=1}^{n} E_{Z \sim \text{אפוסטריורי}}\left[\log\left(\Pr_{\hat{\theta}_t}(X_i = x_i, z_i)\right)\right]$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{k} \left(\Pr_{\hat{\theta}_t}(z_i = j | X_i = x_i)\right) \log\left(\Pr_{\hat{\theta}_t}(X_i = x_i, z_i = j)\right)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{k} a_{i,j}^{t} \left( \log p_j + \sum_{m=1}^{d} x_{i,m} \log q_{j,m} + (1 - x_{i,m}) \log(1 - q_{j,m}) \right)$$

$$\mathcal{L} = Q - \lambda \left( \sum_{j=1}^{k} p_j - 1 \right)$$

$$\frac{\partial \mathcal{L}}{\partial p_j} = \frac{\partial Q}{\partial p_j} - \lambda = \sum_{i=1}^{n} \frac{a_{i,j}^{t} 1}{p_j} - \lambda = 0$$

$$\rightarrow p_j = \frac{1}{\lambda} \sum_{i=1}^{n} a_{(i,j)}^{t}$$

$$\sum_{j=1}^{k} p_j = 1 \rightarrow \sum_{j=1}^{k} \sum_{i=1}^{n} a_{i,j}^{t} = \lambda$$

$$p_j^{t+1} = \frac{\sum_{i=1}^{n} a_{i,j}^{t}}{n}$$

$$\frac{\partial \mathcal{L}}{\partial q_{j,m}} = \frac{\partial Q}{\partial q_{j,m}} = \sum_{i=1}^{n} a_{i,j}^{t} \left( \frac{x_{i,m}}{q_{j,m}} - \frac{1 - x_{i,m}}{1 - q_{j,m}} \right) = 0$$

$$(1 - q_{j,m}) \sum_{i=1}^{n} a_{i,j}^{t} x_{i,m} - q_{j,m} \sum_{i=1}^{n} a_{i,j}^{t} (1 - x_{i,m}) = 0$$

$$\rightarrow q_{j,m} \sum_{i=1}^{n} a_{i,j}^{t} = \sum_{i=1}^{n} a_{i,j}^{t} x_{i,m}$$

$$q_{j,m}^{t+1} = \frac{\sum_{i=1}^{n} a_{i,j}^{t} x_{i,m}}{\sum_{i=1}^{n} a_{i,j}^{t}}$$